Split-Ordered Lists - Lock-free Resizable Hash Tables

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We present the first lock-free implementation of an extensible hash table running on current architectures. Our algorithm provides concurrent insert, delete, and search operations with an expected $O(1)$ cost. It consists of very simple code, easily implementable using only load, store, and compare-and-swap operations. The new mathematical structure at the core of our algorithm is *recursive split-ordering*, a way of ordering elements in a linked list so that they can be repeatedly "split" using a single compare-and-swap operation. Though lock-free algorithms are expected to work best in multiprogrammed environments, empirical tests we conducted on a large shared memory multiprocessor show that even in non-multiprogrammed environments, the new algorithm performs as well as the most efficient known lock-based resizable hash-table algorithm, and in high load cases it significantly outperforms it.

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1. INTRODUCTION

Hash tables, and specifically extensible hash tables, serve as a key building block of many high performance systems. A typical extensible hash table is a continuously resized array of buckets, each holding an expected constant number of elements, and thus requiring an expected constant time for insert, delete and search operations [2]. The cost of resizing, the redistribution of items between old and new buckets, is amortized over all table operations, thus keeping the average complexity of any one operation constant. In this paper, "resizing" means extending the table. It has been shown elsewhere [13] that as a practical matter, hash tables need only increase in size.

We are concerned in implementing the hash table data structure on multiprocessor machines, where efficient synchronization of concurrent access to data structures is essential. Lock-free algorithms have been proposed in the past as an appealing alternative to lock-based schemes, as they utilize strong primitives such as CAS (*compare-and-swap*) to achieve fine grained synchronization. However, lock-free algorithms typicaly require greater design efforts, being conceptually more complex.

This paper presents the first lock-free extensible hash table that works on current architectures, that is, uses only loads, stores and CAS (or LL/SC [23]) operations. In a manner similar to sequential linear hashing [17] and fitting real-time applications, resizing costs are split incrementally to achieve expected $O(1)$ operations per insert, delete and search. It is simple to implement, leading us to hope it will be of interest to practitioners as well as researchers. As we explain shortly, it is based on a novel *recursively split-ordered* list structure. Our empirical testing shows that in a concurrent environment, even without multiprogramming, our lock-free algorithm performs as well as the most efficient known lock-based resizable hash-table algorithm due to Lea [15], and in high load cases it significantly outperforms it.

1.1 Background

There are several lock-based concurrent hash table implementations in the literature. In the early eighties, Ellis proposed an extensible concurrent hash table for distributed data based on a two level locking scheme, first locking a table directory and then the individual buckets [3; 4]. Michael [20] has recently shown that on shared memory multiprocessors, simple algorithms using a reader-writer lock [19] per bucket have reasonable performance for non-resizable tables. However, to resize one would have to hold the locks on all buckets simultaneously, leading to major overheads. A recent algorithm by Lea [15], proposed for *java.util.concurrent*, the Java™ Concurrency Package, is probably the most efficient known extensible hash algorithm. It is based on a more sophisticated locking scheme that involves a small number of high level locks rather than a lock per bucket, and allows concurrent searches while resizing the table, but not concurrent inserts or deletes. In general, lock-based hash-table algorithms are expected to suffer from the typical drawbacks of blocking synchronization: deadlocks, long delays, and priority inversions [6]. These drawbacks become more acute when performing a *resize* operation, an elaborate "global" process of redistributing the elements in all the hash table's buckets among new added buckets. Designing a lock-free resizable hash table is thus a matter of both practical and theoretical interest.

Michael, in [20], builds on the work of Harris [8] to provide an effective compare-and-swap (CAS) based lock-free linked-list algorithm (which we will elaborate upon in the following section). He then uses this algorithm to design a lock-free hash

structure: a fixed size array of hash buckets with lock-free insertion and deletion into each. He presents empirical evidence that shows a significant advantage of this hash structure over lock-based implementations in multiprogrammed environments. However, this structure is not resizable: if the number of elements grows beyond the predetermined size, the time complexity of operations will no longer be constant.

As part of his "two-handed emulation" approach, Greenwald [7] provides a lock-free hash table that can be resized based on a double-compare-and-swap (DCAS) operation. However, DCAS, an operation that performs a CAS atomically on two non-adjacent memory locations, is not available on current architectures. Moreover, Greenwald's hash table is resizable, but is not a true resizable hash table since the average number of steps per operation is not constant: it involves an elaborate "checking" scheme in which every process independently traverses a linear number of buckets to guarantee the lock-free progress property.

Independently of our work, Gao et. al [5] have developed a resizable and "almost wait-free" hashing algorithm based on an open addressing hashing scheme and using only CAS operations. Their algorithm maintains the dynamic size by periodically switching to a global resize state in which multiple processes collectively perform the migration of items to new buckets. They suggest to perform migration using a write-all algorithm [12]. Theoretically, each operation in their algorithm requires more than constant time on average because of the complexity of performing the write-all [12], and so it is not a true resizable hash-table. However, the non-constant factor is small, and the performance of their algorithm in practice will depend on the yet-untested real-world performance of algorithms for the write-all problem [12; 14].

1.2 The Lock-free Resizing Problem

What is it that makes lock-free resizable hashing hard to achieve? The core problem is that even if individual buckets are lock-free, when resizing the table, several items from each of the "old" buckets must be relocated to a bucket among "new" ones. However, in a single CAS operation, it seems impossible to atomically move even a single item, as this requires one to remove the item from one linked list and insert it in another. If this move is not done atomically, elements might be lost, or to prevent loss, will have to be replicated, introducing the overhead of "replication management". The lock-free techniques for providing the broader atomicity required to overcome these difficulties imply that processes will have to "help" others complete their operations. Unfortunately, "helping" requires processes to store state and repeatedly monitor other processes' progress, leading to redundancies and overheads that are unacceptable if one wants to maintain the constant time performance of hashing algorithms.

1.3 Split-Ordered Lists

To implement our algorithm, we thus had to overcome the difficulty of atomically moving items from old to new buckets when resizing. To do so, we decided to, metaphorically speaking, flip the linear hashing algorithm on its head: our algorithm *will not move the items among the buckets*, rather, it *will move the buckets among the items*. More specifically, as shown in Figure 1, the algorithm keeps all the items in one lock-free linked list, and gradually assigns the bucket pointers to the places in the list where a sublist of "correct" items can be found. A bucket is initialized upon first access by assigning it to a new "dummy" node (dashed contour) in the list, preceding all items that should be in that bucket. A newly created

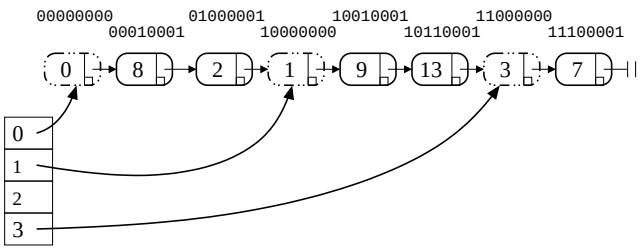


Fig. 1. A Split-Ordered Hash Table

bucket splits an older bucket's chain, reducing the access cost to its items. Our table uses a modulo 2^i hash (there are known techniques for "pre-hashing" before a modulo 2^i hash to overcome possible binary correlations among values [15]). The table starts at size 2 and repeatedly doubles in size.

Unlike moving an item, the operation of directing a bucket pointer can be done in a single CAS operation, and since items are not moved, they are never "lost". However, to make this approach work, one must be able to keep the items in the list sorted in such a way that any bucket's sublist can be "split" by directing a new bucket pointer within it. This operation must be recursively repeatable, as every split bucket may be split again and again as the hash table grows. To achieve this goal we introduced *recursive split-ordering*, a new ordering on keys that keeps items in a given bucket adjacent in the list throughout the repeated splitting process.

Magically, yet perhaps not surprisingly, recursive split-ordering is achieved by simple *binary reversal*: reversing the bits of the hash key so that the new key's most significant bits (MSB) are those that were originally its least significant[1]. In Figure 1 the split-order key values are written above the nodes. The dashed-line nodes are the special dummy nodes corresponding to buckets with original keys that are 0,1,2, and 3 modulo 4. The split-order keys of regular (non-dashed) nodes are exactly the bit-reverse image of the original keys after turning on their MSB (in the example we used 8-bit words). For example, items 9 and 13 are in the "1 mod 4" bucket, which can be recursively split in two by inserting a new node between them.

To *insert* (respectively *delete* or *search* for) an item in the hash table, hash its key to the appropriate bucket using recursive split-ordering, follow the pointer to the appropriate location in the sorted items list, and traverse the list until the key's proper location in the split-ordering (respectively until the key or a key indicating the item is not in the list is found). As we show, because of the combinatorial structure induced by the split-ordering, this will require traversal of no more than an expected constant number of items. A detailed proof appears in Section 3.

We note that our design is modular: to implement the ordered items list, one can use one of several non-blocking list-based set algorithms in the literature. Potential candidates are the lock-free algorithms of Harris [8] or Michael [20], or the obstruction-free algorithms of Valois[2][24] or Luchangco et. al [18]. We chose to base our presentation on the algorithm of Michael [20], an extension of the Harris algorithm [8] that fits well with memory management schemes [9; 21] and performs

[1]As detailed in the next section, some additional bit-wise modifications must be made to make things work properly.

[2]Valois' algorithm was labeled "lock-free" by mistake. It is livelock-prone.

well in practice.

1.4 Complexity

When analyzing the complexity of concurrent hashing schemes, there are two adversaries to consider: one controlling the distribution of item keys, the other controlling the scheduling of thread operations.

As we show in Section 3, if we make the standard assumption of a hash function with a uniform distribution, then under any scheduling adversary our new algorithm provides a lock-free extensible hash table with average $O(1)$ cost per operation. The complexity improves to expected constant time if we assume a *constant extendibility rate*, meaning that the table is never extended (doubled in size) a non-constant number of times while a thread is delayed by the scheduler. Constant expected time is an improvement over average expected time since it means that given a good hash function, the adversary cannot cause any single operation to take more than a constant number of steps.

One feature in which the new algorithm is similar in flavor to sequential linear hashing algorithms [17] (in contrast to all the above algorithms [5; 7; 15]) is that resizing is done incrementally and only bad distributions (ones that have very low probability given a uniform hash function) or extreme scheduling scenarios can cause the cost of an operation to exceed constant time. This makes the algorithm better suited for real-time applications.

1.5 Performance

We tested our new *split-ordered list* hash algorithm versus the most-efficient known lock-based implementation due to Lea [15]. We created an optimized `C++` based version of the algorithm and compared it to split-ordered lists using a collection of tests executed on a 72-node shared memory machine. Though lock-free algorithms are expected to benefit systems especially in multiprogrammed environments, our experiments, presented in Section 4, show that split-ordered lists perform as well as Lea's algorithm even in the less favorable non-multiprogrammed test cases. Under high loads they significantly outperform Lea's algorithm, exhibiting up to four times higher throughput. They also exhibit greater robustness, for example, in experiments where the hash function is biased to create non-uniform distributions.

The remainder of this paper is organized as follows. In the next section we describe the background and the new algorithm in depth. In Section 3 we bring the full correctness proof. In Section 4 the preliminary empirical results are presented and discussed.

2. THE ALGORITHM IN DETAIL

Our hash table data structure consists of two interconnected sub-structures (see Figure 1): a linked list of nodes containing the stored items and keys, and an expanding array of pointers into the list. The array entries are the logical "buckets" typical of most hash tables. Any item in the hash table can be reached by traversing down the list from its head, while the bucket pointers provide shortcuts into the list in order to minimize the search costs per item.

The main difficulty in maintaining this structure is in managing the continuous coverage of the full length of the list by bucket pointers as the number of items in the list grows. The distribution of bucket pointers among the list items must remain dense enough to allow constant time access to any item. Therefore, new buckets need to be created and assigned to sparsely covered regions in the list.

The bucket array initially has size 2, and is doubled every time the number of items in the table exceeds $size \cdot L$, where L is a small integer denoting the *load factor*, the maximum number of items one would expect to find in each logical bucket of the hash table. The initial state of all buckets is *uninitialized*, except for the bucket of index 0, which points to an empty list, and is effectively the head pointer of the main list structure. Each bucket goes through an initialization procedure when first accessed, after which it points to some node in the list.

When an item of key k is inserted, deleted, or searched for in the table, a hash function modulo the table size is used, i.e. the bucket chosen for item k is $k \ mod \ size$. The table size is always equal to some power 2^i, $i \geq 1$, so that the bucket index is exactly the integer represented by the key's i least significant bits (LSBs). The hash function's dependency on the table *size* makes it necessary to take special care as this size changes: an item that was inserted to the hash table's list before the resize must be accessible, after the resize, from both the buckets it already belonged to and from the new bucket it will logically belong to given the new hash function.

2.1 Recursive split-ordering

The combination of a modulo-size hash function and a 2^i table size is not new. It was the basis of the well known sequential resizable Linear Hashing scheme proposed by Litwin [17], was the basis of the two-level locking hash scheme of Ellis [3], and was recently used by Lea in his concurrent resizable hashing scheme [15]. The novelty here is that we use it as a basis for a combinatorial structure that allows us to repeatedly "split" all the items among the buckets without actually changing their position in the main list.

When the table size is 2^i, a logical table bucket b contains items whose keys k maintain $k \ mod \ 2^i = b$. When the size becomes 2^{i+1}, the items of this bucket are split into two buckets: some remain in the bucket b, and others, for which $k \ mod \ 2^{i+1} = b + 2^i$, migrate to the bucket $b + 2^i$. If these two groups of items were to be positioned one after the other in the list, splitting the bucket b would be achieved by simply pointing bucket $b + 2^i$ after the first group of items and before the second. Such a manipulation would keep the items of the second group accessible from bucket b as desired.

Looking at their keys, the items in the two groups are differentiated by the i'th binary digit (counting from right, starting at 0) of their items' key: those with 0 belong to the first group, and those with 1 – to the second. The next table doubling will cause each of these groups to split again into two groups differentiated by bit $i + 1$, and so on. This process induces *recursive split-ordering*, a complete order on keys, capturing how they will be repeatedly split among logical buckets. Given a key, its order is completely defined by its bit-reversed value.

Let us now return to the main picture: an exponentially growing array of (possibly uninitialized) buckets maps to a linked list ordered by the split-order values of inserted items' keys, values that are derived by reversing the bits of the original keys. Buckets are initialized when they are accessed for the first time. List operations such as `insert`, `delete` or `find` are implemented via a linearizable lock-free linked list algorithm. However, having additional references to nodes from the bucket array introduces a new difficulty: it is non-trivial to manage deletion of nodes pointed to by bucket pointers. Our solution is to add an auxiliary dummy node per bucket, preceding the first item of the bucket, and to have the bucket pointer point to this dummy node. The dummy nodes are not deleted, which helps

```
so_key_t so_regularkey(key_t key) {
   return REVERSE(key OR 0x8000...0000);
}

so_key_t so_dummykey(key_t key) {
   return REVERSE(key)
}
```

Fig. 2. The Split-Ordering Transformation

us keep things simple.

In more detail, when the table size is 2^{i+1}, the first time bucket $b+2^i$ is accessed, a dummy node is created, holding the key $b+2^i$. This node is inserted to the list via bucket b, the *parent* bucket of $b+2^i$. Under split-ordering, $b+2^i$ precedes all keys of bucket $b+2^i$, since those keys must end with $i+1$ bits forming the value $b+2^i$. This value also succeeds all the keys of bucket b that do not belong to $b+2^i$: they have identical i LSBs, but their bit numbered i is "0". Therefore, the new dummy node is positioned in the exact location in the list that separates the items that belong to the new bucket from other items of bucket b. In order to distinguish dummy keys from regular ones we set the most significant bit of regular keys to "1", and leave the dummy keys with "0" at the MSB. Figure 2 defines the complete split-ordering transformation[3].

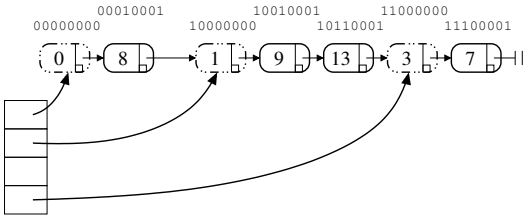
Figure 3 describes a bucket initialization caused by an insertion of a new key to the set. The insertion of key 10 is invoked when the table size is 4 and buckets 0,1 and 3 are already initialized.

2.2 The Continuously Growing Table

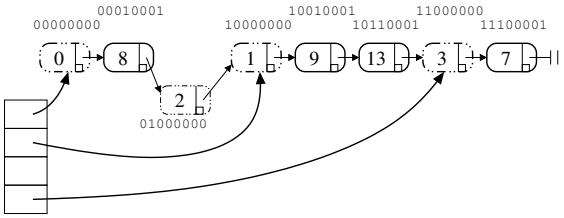
We can now complete the presentation of our algorithm. We use the lock-free ordered linked-list algorithm of Michael [20] to maintain the main linked list with items ordered based on the split-ordered keys. This algorithm is an improved variant, including improved memory management, of an algorithm by Harris [8]. Our presentation will not discuss the various memory reclamation options of such linked-list schemes, and we refer the interested reader to [8; 9; 20; 21]. To keep our presentation self contained, we provide in Appendix A the code of Michael's linked list algorithm. This implementation is linearizable, implying that each of these operations can be viewed as happening atomically at some point within its execution interval.

Our algorithm decides to double table size based on the average bucket load. This load is determined by maintaining a shared counter that tracks the number of items in the table. The final detail we need to deal with is how the array of buckets is repeatedly extended. To simplify the presentation, we keep the table buckets in one continuous memory segment as depicted in Figure 4. This approach is somewhat impractical, since table doubling requires one process to reallocate a very large memory segment while other processes may be waiting. The practical version of this algorithm, which we used for performance testing, actually employs an additional level of indirection for accessing buckets: a main array points to segments of buckets, each of which is a bucket array. A segment is allocated only

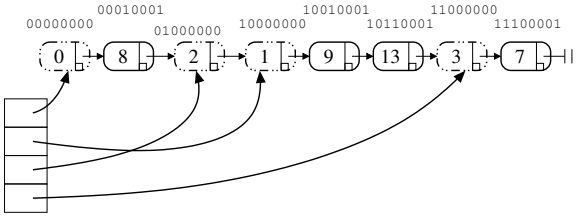
[3] An efficient implementation of the `REVERSE` function utilizes a 2^8 or 2^{16} lookup table holding the bit-reversed values of $[0..2^8-1]$ or $[0..2^{16}-1]$ respectively.



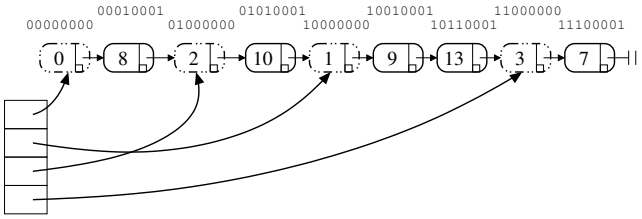
(a) Buckets 0,1 and 3 are initialized. Bucket 2 is uninitialized



(b) Insert(10) is invoked, requiring bucket 2's initialization.
A new dummy node is inserted, with split–order key of 2.



(c) Bucket 2 is assigned to the new dummy node



(d) The split–order regular key 10 is inserted to bucket 2

Fig. 3. Insertion into the split-ordered list

on the first access to some bucket within it. The code for this dynamic allocation scheme appears in Section 2.4.

Finally, the reader may have noticed that based on the approach described above, when an uninitialized bucket is accessed in a table of size *size*, one might need to recursively initialize (i.e. split) all $O(\log size)$ of its parent buckets to allow insertion of a new item. Though the total complexity in such a case is logarithmic, not constant, our algorithm still works. This is because given a uniform distribution of items, the chances of the above scenario happening are low, and in fact, the expected length of such a bad sequence of parent initializations is constant.

```
struct MarkPtrType {
  <mark, next>: <bool, NodeType *>
};

struct NodeType {
  so_key_t key;
  MarkPtrType <mark, next>;
};

MarkPtrType* T[ ]; // buckets
unsigned int count; // total item count
unsigned int size; // current table size

/* thread-private variables */
MarkPtrType *prev;
MarkPtrType <pmark, cur>;
MarkPtrType <cmark, next>;
```

Fig. 4. Types and Structures

2.3 The Code

We now provide the code of our algorithm. Figure 4 specifies some type definitions and global variables. The accessible shared data structures are the array of buckets `T`, a variable `size` storing the current table size, and a counter `count` denoting the number of `regular` keys currently inside the structure. The counter is initially 0, and the buckets are set as *uninitialized*, except the first one, which points to a node of key 0, whose `next` pointer is set to `NULL`. Three private variables serve each one of the running threads: `prev`, `cur` and `next`. Those variables have the same functionality as in Michael's algorithm [20], as they are set by `list_find` to point at the nodes around the searched key, and subsequently used by the same thread inside other functions. In Figure 5 we show the implementation of the `insert`, `search` and `delete` operations. The `fetch-and-inc` operation is implemented in a lock-free manner via a simple repeated loop of CAS operations, which as we show, given the low access rates, has a negligible performance overhead.

The function `insert` creates a new node and assigns it a split-order key. The bucket is computed as `key` *mod* `size`. If the bucket has not been initialized yet, `initialize_bucket` is called. Then, the node is inserted to the bucket by using `list_insert`. If the insertion is successful, one can proceed to increment the item count using a `fetch-and-inc` operation (`fetch-and-inc` can be implemented in a lock-free manner [22]). A check is then performed to test whether the load factor has been exceeded. If so, the table size is doubled, causing a new segment of uninitialized buckets to be appended.

The function `search` ensures that the appropriate bucket is initialized, and then calls `list_find` on `key` after marking it as regular and inverting its bits. `list_find` ceases to traverse the chain when it encounters a node containing a higher or equal (split-ordered) key. Notice that this node may also be a dummy node marking the beginning of a different bucket.

The function `delete` also makes sure that the `key`'s bucket is initialized. Then it calls `list_delete` to delete `key` from its bucket after it is translated to its split-order value. If the deletion succeeds, an atomic decrement of the total item count is performed.

The role of `initialize_bucket` is to direct the pointer in the array cell of the index `bucket`. The value assigned is the address of a new dummy node containing the dummy key `bucket`. First, the dummy node is created and inserted to an existing bucket, `parent`. Then the cell is assigned the node's address. If the parent bucket is not initialized, the function is called recursively with `parent`. In order to control the recursion we maintain the invariant that `parent`$<$ `bucket`. It is also wise to choose `parent` to be as close as possible to `bucket` in the list, but still preceding it. Formally, the following constraints define the algorithm's choice of

```
int insert(KeyType key) {
I1:  node = new_node(so_regularkey(key));
I2:  bucket = key % size;
I3:  if (T[bucket] == UNINITIALIZED)
I4:    initialize_bucket(bucket);
I5:  if (!list_insert(&(T[bucket]), node)) {
I6:    delete_node(node);
I7:    return 0;
     }
I8:  csize = size;
I9:  if (fetch-and-inc(&count) / csize > MAX_LOAD)
I10:   CAS(&size, csize, 2 * csize);
I11: return 1;
}

int search(KeyType key) {
S1:  bucket = key % size;
S2:  if (T[bucket] == UNINITIALIZED)
S3:    initialize_bucket(bucket);
S4:  return list_find(&(T[bucket]),
                      so_regularkey(key));
}

int delete(KeyType key) {
D1:  bucket = key % size;
D2:  if (T[bucket] == UNINITIALIZED)
D3:    initialize_bucket(bucket);
D4:  if (!list_delete(&(T[bucket]),
                so_regularkey(key)))
D5:    return 0;
D6:  fetch-and-dec(&count);
D7:  return 1;
}

void initialize_bucket(uint bucket) {
B1:  parent = GET_PARENT(bucket);
B2:  if (T[parent] == UNINITIALIZED)
B3:    initialize_bucket(parent);
B4:  dummy = new node(so_dummykey(bucket));
B5:  if (!list_insert(&(T[parent]), dummy)) {
B6:    delete dummy;
B7:    dummy = cur;
     }
B8:  CAS(&(T[bucket]), UNINITIALIZED, dummy);
}
```

Fig. 5. Our split-order based hashing algorithm

parent uniquely (the split-order is denoted by $\prec$):

$$\forall k \neq \texttt{parent}, k \prec \texttt{bucket} \Rightarrow \texttt{parent} \succ k$$
$$\texttt{parent} \prec \texttt{bucket}$$
$$\texttt{parent} < \texttt{bucket}$$

This value is achieved by unsetting bucket's most significant turned-on bit. If the exact dummy key already exists in the list, it may be the case that some other process tried to initialize the same bucket, but for some reason has not completed the second step. In this case, list_insert will fail, but the private variable cur will point to the node holding the dummy key. The newly created dummy node can be freed and the value of cur used.

As we will show in the proof, traversing the list through the appropriate bucket and dummy node will guarantee the node matching a given key will be found, or declared not-found in an expected constant number of steps.

2.4 Dynamic Sized Array

Our presentation so far simplified the algorithm by keeping the buckets in one continuous memory segment. This approach is somewhat impractical, since table doubling requires one process to reallocate a very large memory segment while other processes may be waiting. In practice, we avoid this problem by introducing an additional level of indirection for accessing buckets: a "main" array points to segments of buckets, each of which is a bucket array. A segment is allocated only on the first access to some bucket within it.

Applying this variation is done by defining T as an array of bucket segments, and accessing the table by calls to get_bucket and set_bucket as defined in Figure 6.

```
typedef MarkPtrType[SEGMENT_SIZE] segment_t;
segment_t T[ ];

MarkPtrType * get_bucket(T, bucket) {
  segment = bucket / SEGMENT_SIZE;
  if (T[segment] == NULL)
    return UNINITIALIZED;
  return &T[segment][bucket % SEGMENT_SIZE];
}

void set_bucket(T, bucket, head) {
  segment = bucket / SEGMENT_SIZE;
  if (T[segment] == NULL) {
    new_segment = new segment_t;
    new_segment[0..SEGMENT_SIZE-1] =
      UNINITIALIZED;
    if (!CAS(&T[segment], NULL, new_segment))
      free(new_segment);
  }
  T[segment][bucket % SEGMENT_SIZE] = head;
}
```

Fig. 6. Dynamic Sized Array

3. CORRECTNESS PROOF

This section contains a formal proof that our algorithm has the desired properties of a resizable hash table. Our model of multiprocessor computation follows [11], though for brevity, we will use operational style arguments.

Our linearizable hash table data structure implements an abstract *set* object in a lock-free way so that all operations take an expected constant number of steps on average. Our correctness proof will thus have to prove that our concurrent implementation is linearizable to a sequential set specification, that it is lock-free, and that given a "good" class of hash functions, all operations take an expected constant number of steps on average.

3.1 Correct Set Semantics

We begin by proving that the algorithm complies with the abstract set semantics. We use the sequential specification of a "dynamic set with dictionary operations" as defined in [2], including the three functions *insert, delete* and *search*. The *insert* operation returns 1 if the key was successfully inserted to the set, and 0 if that key already existed in the table. The *search* operation returns 1 if the key is in the set, 0 otherwise. The *delete* operation returns 1 if the key was successfully deleted from the set and 0 if it was not found.

Given a sequential specification of a set, our proof will provide specific linearization points mapping operations in our concurrent implementation to sequential operations so that the histories meet the specification.

Let *list* refer to the non-blocking ordered linked list of all items, pointed to by the buckets of the hash table. Execution histories of our algorithm include sequences of `list_find`, `list_insert`, and `list_delete` operations on this list. Though we argue about these as operations on the shared *list* and not as abstract set operations, our proof will treat these operations as atomic operations. This is a valid approach since they are linearizable by definition of the list-based set algorithms [8; 20]. We do however need to make additional claims about properties of operations on the *list*, since we will apply them to various "midpoints" pointed to by buckets, and not only to the start of the list as in the original use of these algorithms of [8; 20]. To this end we present the following invariant which refers to the structure of the list in any state in the execution history of our algorithm.

INVARIANT 1. *In any state:*

—*all keys in the list starting at* `T[0]` *are sorted in an ascending order.*

—*for every* $0 \leq i < size$ *if* `T[i]` *is initialized, then the node pointed by* `T[i]` *holds the key* `so_dummykey[i]` *and is reachable from* `T[0]` *by traversing the list following the nodes' next pointers.*

PROOF. Initially, the invariant holds. We will show that every operation that modifies the data structure preserves the invariant. Lines I9 and D6 manipulate the shared counter, but have no impact on the invariant. Line I10 doubles `size`, which adds news buckets, but since `size` only grows, those new buckets are uninitialized, and the invariant is unaffected.

Assuming that the invariant is true just before line I5, we will show that it is preserved. If `list_insert` fails, the shared state has not changed. Otherwise, we use the induction assumption that `T[bucket]` points to a node holding the key `so_dummykey(bucket)`, and that `node` is in the list beginning at `T[0]`. The procedure `list_insert` inserts `node` to the list `T[bucket]`. This trivially preserves

the second condition of the invariant for the bucket. The new node's key is the bit reverse of `key OR 0x800...0`. The array index `bucket` has the same $\log size$ less significant bits as `key`, and all of its remaining bits are 0. Therefore, the new node's key is ordered after the first node of `T[bucket]`, whose key is the bit reverse of `bucket`. The first part is also preserved, that is, the list reachable from `T[0]` remains sorted since all keys before `T[bucket]` are by the inductive assumption ordered and have lower keys than `so_dummykey(bucket)` and so are properly positioned before the new node, and all other keys are positioned properly by the inductive assumption and the correctness of the `list_insert` operation, since they are a part of the list pointed to by `T[bucket]`.

The `list_delete` operation of line D4 only deletes a key, and thus cannot affect the order. The deleted `node` cannot be the first node of `T[bucket]`, since the least significant bit of its key is 0 and the deleted key's least significant bit is 1.

The function `list_insert` in line B5 inserts a `node` with key `so_dummykey(bucket)` to the sublist `T[parent]`, starting with a node holding `so_dummykey(parent)`. The key `parent` is defined by turning off the index `bucket`'s most significant "1" bit, so the insertion is not before the first node of the sublist starting at `T[parent]`, and as in the above proof for the case of I5, the invariant is preserved.

Finally, the CAS at B8 sets `T[bucket]` to either the dummy node created at B4, or the one assigned at B7. In the first case, since a dummy node created in line B4 is inserted, the second condition of the invariant follows immediately from the correctness of the `list_insert` operation. The first condition follows since the dummy node is inserted in order *after* its parent node which is necessarily ordered before it. In the second case, `list_insert` failed because the key `so_dummykey(bucket)` was in the list and `cur` was by the definition of *list_insert* set to the node holding that key, so both parts of the invariant follow. □

We now define the set H of keys whose items are in the hash table in any given state.

Definition 1. For any pointer p, let $S(p)$ be the set of keys in the sorted linked list beginning with the pointer p. Let the *hash table set*

$$H = \{\texttt{k} \mid \texttt{so\_regularkey(k)} \in S(\texttt{T[0]})\}$$

The set H defines the abstract state of the table. For each one of the hash table operations, we will now show that one can pick a linearization point within its execution interval, so that at this point it has modified the abstract state, that is, the set H, according to the specified operation's semantics. Specifically, we will choose the following linearization points:

—the `insert` operation is linearized in line I5, at the `list_insert` operation,
—the `search` operation is linearized in line S4, at the `list_find` operation, and
—the `delete` operation is linearized in line D4, at the `list_delete` operation.

We start with the following helpful lemma.

Lemma 1. *In lines I5, S4, and D4,* `T[bucket]` *is already initialized, and at B5* `T[parent]` *is already initialized.*

Proof. All of the lines above follow a validation that `T[bucket]` is initialized. If `T[bucket]` is not initialized, `initialize_bucket` is called, and `initialize_bucket` must execute line B8 before it returns. In line B8, if the `bucket` is uninitialized, the CAS succeeds and the `bucket` initialization is executed. □

Note that in the proof above we were not interested in whether the initialization sequence (where initializing a bucket causes initialization of the parent) actually terminates, but rather that if it did terminate then all parents of a bucket were initialized.

LEMMA 2. *If* `key` *is in* H *in line I5, then* `insert` *fails and if it is not,* `insert` *succeeds and* `key` *joins* H.

PROOF. If `key` is in H, `so_regularkey(key)` $\in S($`T[0]`$)$. According to Lemma 1, `T[bucket]` is initialized, and using Invariant 1 we conclude that the node pointed by `T[bucket]` has the key `so_dummykey(bucket)` and it is a part of the list. The list is sorted, and

$$\begin{gathered}\texttt{so\_dummykey(bucket)} = \texttt{REVERSE(bucket)} = \\ \texttt{REVERSE(key mod size)} < \texttt{REVERSE(key OR 0x800..0)} = \\ \texttt{so\_regularkey(key)}.\end{gathered} \tag{1}$$

Thus, the searched key is in the sublist, $S($`T[bucket]`$)$. The `list_insert` at I5 will fail and so will `insert`. If `key` is not in H, it is also not in $S($`T[bucket]`$)$, and `list_insert` inserts `so_regularkey(key)` in the `bucket`'s sublist. From that state on, `so_regularkey` $\in S($`T[0]`$)$, i.e. `key` is in H. □

LEMMA 3. *If* `key` *is in* H *at line S4, the* `search` *succeeds, and otherwise the* `search` *fails.*

PROOF. If when line S4 is executed `key` is in H, then `so_regularkey(key)` is in $S($`T[0]`$)$. `T[bucket]` is assigned to a node in that list, holding the key `so_dummykey(bucket)`. Using Equation 1, we conclude that the searched key is in $S($`T[bucket]`$)$, so `list_find` succeeds an so does `search`. If in line S4 `key` is not in H, it cannot be in $S($T[bucket]$)$, so `list_find` fails. □

LEMMA 4. *If* `key` *is in* H *in line D4,* `delete` *succeeds and removes* `key` *from* H*, and otherwise* `delete` *fails.*

PROOF. If `key` is in H, then `so_regularkey(key)` is in $S($`T[0]`$)$. `T[bucket]` is assigned to a node inside that list, and this node is holding the key `so_dummykey(bucket)`. Using Equation 1, we conclude that the searched key is in $S($`T[bucket]`$)$, so `list_delete` removes it. If `key` is not in H, it cannot be in $S($`T[bucket]`$)$, so `list_delete` fails. □

From Lemma 2, Lemma 3, and Lemma 4 it follows that:

THEOREM 1. *The split-ordered list algorithm of Figure 5 is a linearizable implementation of a set object.*

3.2 Lock Freedom

Our algorithm uses loads and stores together with implementations of a list-based set and a shared counter as primitive objects/operations. As we will show, in terms of these primitive operations the algorithm's implementation is wait-free, that is, each thread always completes in a finite number of operations. This implies that its overall progress condition in terms of primitive machine operations will be exactly that of the underlaying implementation of these objects. Since in this presentation we used use as building blocks the lock-free list-based sets of [8; 20] and a lock-free shared counter, our implementation will also be lock-free. As noted in the

introduction, in some cases there are advantages in using the obstruction free list-based set algorithm of [18]. If [18] is used together with a lock-free shared counter, our hash table will be obstruction free [10].

THEOREM 2. *The split-ordered list algorithm of Figure 5 is a wait-free implementation of a set object in terms of* `load`, `store`, `fetch-and-inc`, `fetch-and-dec`, `list_find`, `list_insert` *and* `list_delete` *operations.*

PROOF. The functions `insert`, `search`, `delete` and `initialize_bucket` all take a finite number of steps, each of which is a machine level `load` or `store` operation or an operation on the list based set object or the shared counter. The `initialize_bucket` procedure is the only one with a recursive call. However, the recursion of `initialize_bucket` is limited, since each step is executed on the `parent` of a `bucket`, which satisfies `parent` $<$ `bucket`. Since `bucket` 0 is initialized from the start, the recursion is finite, and the implementation is wait-free. □

The lock-freedom property means that a thread executing the hash table operation completes in a finite number of steps unless other threads are infinitely making progress. Thus, it is a weaker requirement than wait-freedom, and by combining implementations the following is a corollary of Theorem 2:

COROLLARY 1. *The split-ordered list algorithm of Figure 5 with lock-free implementations of* `list_find`, `list_insert`, `list_delete`, `fetch-and-inc`, `fetch-and-dec` *operations is lock-free.*

COROLLARY 2. *The split-ordered list algorithm of Figure 5 with obstruction-free implementations of* `fetch-and-inc`, `fetch-and-dec`, `list_find`, `list_insert` *and* `list_delete` *operations is obstruction-free.*

The `fetch-and-inc` and `fetch-and-dec` operations have known lock-free implementations [22].

3.3 Complexity

The most important property of a hash table is its expected constant time performance. When analyzing the complexity of hashing in a concurrent environment there are two adversaries one needs to consider: one controlling the distribution of hash values of keys by the hash function (i.e. how good is the hash), the other controlling the scheduling of thread operations. We will follow the standard practice of modelling the hash function as a uniform distribution over keys [2].

We will use the term *expected time* to mean the expected number of machine instructions in the worst case scheduling scenario, assuming a hash function of uniform distribution. The term *average time* will refer to the average number of machine instructions an operation takes in the average case scheduling scenario, also assuming uniform hashing.

We will show that under any scheduling adversary, our algorithm performs all hash table operations in constant average time. The complexity improves to expected constant time if we assume a *constant extendibility rate*, meaning that the table is never extended a non-constant number of times while a thread is delayed by the scheduler. This is an improvement since it means that given a good hash function, the adversary cannot cause any single operation to take more than a constant number of steps unless it delays its progress through more than a constant numbers of global resize operations. Formally, when there are n items in the data structure, a thread must complete a single operation before $n \cdot 2^{f(n)}$ successful insertions of

elements by other threads were completed, where $f(n) \in \omega(1)$. We believe this is the common situation in practice.

Two algorithmic issues require a detailed proof: one is the complexity of list operations, which is essentially the complexity of executing a `list_find`, and the other is the complexity of `initialize_bucket`, which involves recursive calls.

Denote by n the total number of items in the table, and by s the number of buckets, which is assumed to be larger than the number of threads. Let L denote the load factor `MAX_LOAD` in our code, typically a small constant.

LEMMA 5. *For any number p of threads, at all times the following condition holds:*

$$\frac{n-p}{s} \leq L$$

PROOF. Focus on the successful completed `insert` and `delete` operations. Each successful insertion incremented `count` by 1, and each successful deletion decremented it. In any state there are no more than p concurrent operations. Every one of the "already completed" insert operations checked, when executing line I9, that the ratio of `count` and `csize` is not more than L, and doubled the `size` if the gap was exceeded. There are no more than p currently executing `insert` operations, so in a state a resize must be executed since $n/s > L$, no more than p new keys can be inserted to the data structure before the resize takes place. □

LEMMA 6. *Assuming a hash function of uniform distribution, the probability that a bucket is not accessed during the time where the table size $s > p$, is bounded by $e^{-L/2}$.*

PROOF. Focus on a growing table, from size $s/2$ to s and then to $2s$. According to Lemma 6, in the state in which line I10 doubled the table from $s/2$ to s, the number of items in the table was less or equal to $p + Ls/2$. When later in line I10 the table doubled in size to $2s$, the condition of line I9 implies that the number of items was at least Ls. The last two observations imply that during the set of states in which `size` was s, the item count increased in at least $Ls/2 - p$, i.e. line I9 was executed at least $Ls/2 - p$ times. Considering at most p processes that began the insert operation when `size` was less than s, during the same period line I2 was executed at least $Ls/2 - 2p$ times.

Assuming a uniform distribution of the keys, the probability of a bucket b not to be accessed during this period is at most $(\frac{s-1}{s})^{Ls/2-2p}$, asymptotically equal to $e^{-L/2}$. □

LEMMA 7. *For any key k, when the table size is s and the bucket k mod size is initialized, there is no dummy node with key d such that k mod size $\prec d \prec k$, that is, d's split-order is between those of k mod size and k.*

PROOF. Assume by way of contradiction that d is the key of a node such that: k *mod size* $\prec d \prec k$. It is the case that $d < size$ because d is in the list, and bucket indices are always smaller than the table size. Therefore, d has less than $\log_2(size)$ non-zero bits. The keys k and k *mod size* have at least $\log_2(size) - 1$ identical less significant bits. Key $d's$ split-order value is between them, so it must have the same low $log_2(size) - 1$ bits, that actually constitute all of its non-zero bits. This implies that $d = k$ *mod size* under the split-order, a contradiction to the assumption that $d \succ k$ *mod size*. □

LEMMA 8. *If the hash function distributes the keys uniformly,*

—*In any execution history, the list traversal of* `list_find` *takes constant time on average.*

—*Under the constant extendibility rate assumption, the traversal of* `list_find` *takes expected constant time.*

PROOF. For a table of size s, the expected number of uninitialized buckets among the first $s/2$ buckets is no more than $s/2 \cdot e^{-L/2}$, by Lemma 6. For each of the initialized buckets, there is a dummy node in the list holding the bucket index as the split-order value. Therefore, there are at least $s/2 \cdot (1 - e^{-L/2})$ dummy nodes with keys from $0..s/2 - 1$. Those values divide the integer range to $s/2$ equal segments, while the missing items are distributed evenly. There are on average less than

$$\frac{n}{s/2 \cdot (1 - e^{-L/2})} \leq \frac{Ls + p}{s/2 \cdot (1 - e^{-L/2})} = \frac{2L + 2p/s}{1 - e^{-L/2}} \tag{2}$$

nodes between every two dummy nodes. The operation `list_find` is called to search for a key k from the bucket k *mod size*, so using Lemma 7 we conclude that in the state in which it was called there were no dummy nodes between the bucket's dummy node and the node at which the search would be completed. We have just computed that dummy nodes are distributed each $\frac{2L+2p/s}{1-e^{-L/2}}$ nodes, implying that if the table size does not change, the search will take no more than a constant expected number of steps.

We will now show that if the search took more than constant time, there were enough successful inserts to maintain a constant number of steps on average. If `list_find` took $\Omega(r)$ steps, $\Omega(r)$ dummy nodes must have been traversed, since at any time the expected distance between them is constant. All of these dummy nodes were inserted to the list after `list_find` started. The number of dummy nodes in the original bucket doubles each time the table is extended, so there were $\Omega(\log r)$ table resize events. Since there were exactly n items in the table when the `list_find` operation started, the number of items had to rise by $\Omega(rn)$, i.e. $\Omega(rn)$ successful insert operations were completed. There can be no more than p delayed threads, causing overhead of $O(r)$ steps, while $\Omega(rn)$ successful operations complete. The amortized effect is of $O(rp/rn) = O(p/n)$ time, which is constant. Under the constant extendibility rate assumption, $rn < 2^{f(n)} \cdot n$, implying that $\log r$ is constant, and so is r. □

LEMMA 9. *Given a hash function with expected uniform distribution, the number of steps performed by the function* `initialize_bucket` *is constant on average. Under the constant extendibility rate assumption, the complexity is expected constant time.*

PROOF. Recursive calls to `initialize_bucket` terminate when the parent bucket is initialized. To have m recursive calls, m uninitialized ancestor buckets are needed. Applying Lemma 6, this may happen with probability less than $e^{-L(m-1)/2}$, making the expected number of recursive calls constant. By Lemma 8, the `list_insert` call inside `initialize_bucket` costs a constant number of steps on average. If we assume constant extendibility rate (threads are not delayed while the table is doubled a non-constant number of times), a recent ancestor of every bucket is always initialized, and the recursion depth is constant. Also, according to Lemma 8, the execution of `list_insert` is of expected constant time. □

THEOREM 3. *Given a hash function with expected uniform distribution, all hash table operations complete within a constant number of steps on average. Assum-*

ing a constant extendibility rate, all hash table operations complete within expected constant time.

PROOF. Beside executing a constant number of simple instructions, all hash operations call a list traversing routine twice at most (actually only `hash_delete` may cause `list_find` to run twice). By Lemma 8 the list traversals cost a constant average number of steps, and by Lemma 9 the `initialize_bucket` operation also completes within a constant average number of steps. Both of the above lemmas imply that under the constant extendibility rate assumption, the time complexity is constant in the worst case execution assuming a uniform distribution. □

4. PERFORMANCE

We ran a series of tests to evaluate the performance of our lock-free algorithm. Since our algorithm is the first lock-free resizable hash table, it needs to be proven efficient in comparison to existing lock-based resizable hash algorithms. We have thus chosen to compare our algorithm to the resizable hash table algorithm of Lea [15] (revision 1.3), originally suggested as a part of *util.concurrent.ConcurrentHashMap*, the proposed Java™ Concurrency Package, JSR-166.

Lea's algorithm is based on an exponentially growing table of buckets, doubled when the average bucket load exceeds a given load factor. Access to the table buckets is synchronized by 32 locks (on default), dividing the bucket range to 32 interleaved regions, i.e. lock i is obtained when bucket b is accessed if $b \; mod \; 32 = i$. `Insert` and `delete` operations always acquire a lock, but `find` operations are first attempted without locking, and retried with locking upon failure. When a process decides to resize the table, it locks all 32 locks, allocates a larger array and rehashes the buckets' items to their new buckets, utilizing the simplicity of power-of-two hashing. This scheme offers good performance, in comparison to simpler schemes that separately lock each bucket, by significantly reducing the number of locks that need to be acquired when resizing. Figure 11 illustrates the effect of different concurrency levels on Lea's algorithm performance.

We translated the Java™ programming language code by Lea to C++ and simplified it to handle integer keys that also serve as values, exactly as in our new algorithm's code. There is in this algorithm a trade-off, the more locks used, the lower the contention on them, but the higher the global delay when resizing. We thus ran an experiment to confirm that in the translated algorithm there is no significant advantage to using more or less than 32 locks as originally chosen by Lea.

We compared our split-ordered hashing algorithm to Lea's algorithm using a collection of experiments on a 72 node Sun Fire™ 15K, a cache-coherent NUMA machine formed from 18 boards of four 900MHz UltraSPARC® processors, connected by an 18×18 crossbar. The C/C++ code was compiled with a Sun *cc* compiler 5.3, with the flags `-xO5` and `-xarch=v8plusa`.

Lea's algorithm has significant vulnerability in multiprogrammed environments since whenever the resizing processor is swapped out or delayed, the algorithm as a whole grinds to a halt. The significant latency overhead while resizing would also make it less of a fit for real-time environments. However, our tests here are designed to compare the performance of the algorithms in the currently more common environments without multiprogramming or real-time requirements.

We ran a series of experiments measuring the change in throughput as a function of concurrency under various synthetic distributions of *insert*, *delete* and *find*. We

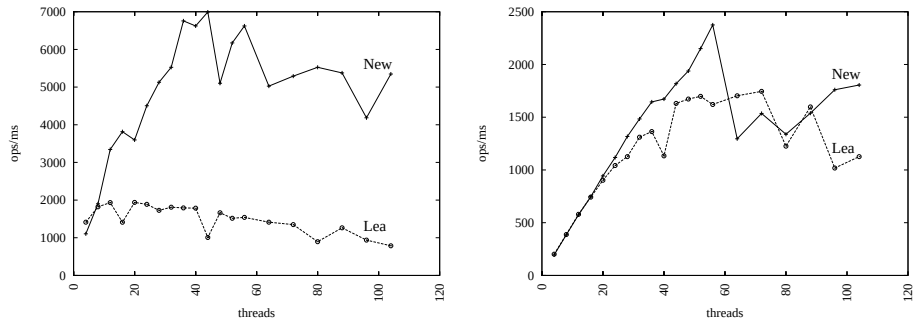


Fig. 7. Throughput with `work` = 0 (left) and `work` = 500 (right)

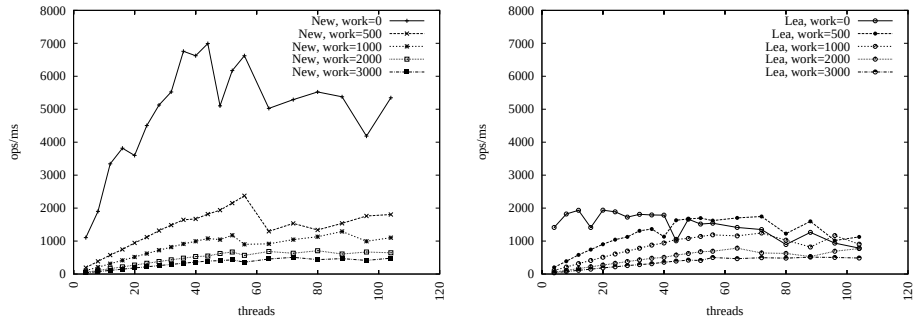


Fig. 8. Throughput of the new algorithm (left) and Lea's algorithm (right)

also varied the workload `work` in various tests by choosing a delay uniformly at random from [0 . . . work].

To capture performance under typical hash-table usage patterns [16] we first look at a mix that consists of about 88% *find* operations, 10% *insert*s and 2% *delete*s. Our first graph, in Figure 7, shows the results of comparing the algorithms under such a pattern. The hash table load factor (the number of items per bucket) for both tested algorithms was chosen as 3. In the presented graph we show the change in throughput as a function of concurrency. As can be seen, at high loads the lock-free split-ordered hashing algorithm significantly outperforms Lea's algorithm at all concurrency levels. In the right hand side of Figure 7, we show the performance of both algorithms when the threads perform a (bounded) random amount of work between operations. Figure 8 illustrates the behavior of both algorithms separately, emphasizing the effect of work between operations.

—When `work` = 0, Lea's algorithm reaches peak performance at about 20 threads and at the same concurrency level, our new algorithm has two times higher throughput. When `work` = 500, Lea's algorithm performs slightly worse. Both algorithms exhibit an almost linear speedup up to 60 threads.

—Our algorithm reaches peak performance at 44 threads when `work` = 0, where it is almost four times faster than Lea's. When `work` = 500, it peaks at 56 threads, having 30% higher throughput.

—Our algorithm's performance fluctuates after reaching peak performance because

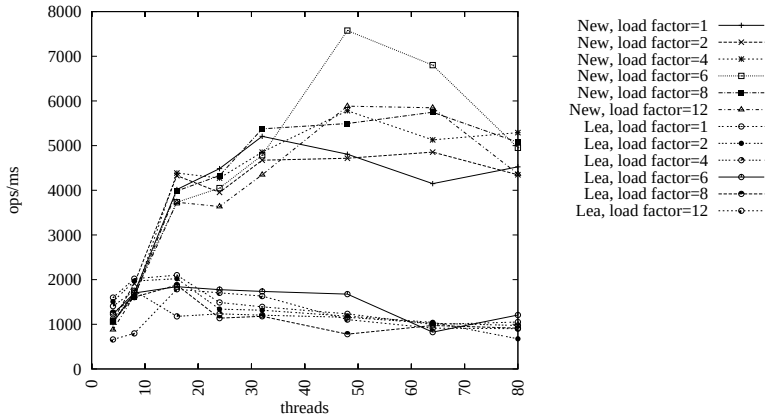


Fig. 9. Varying load factor

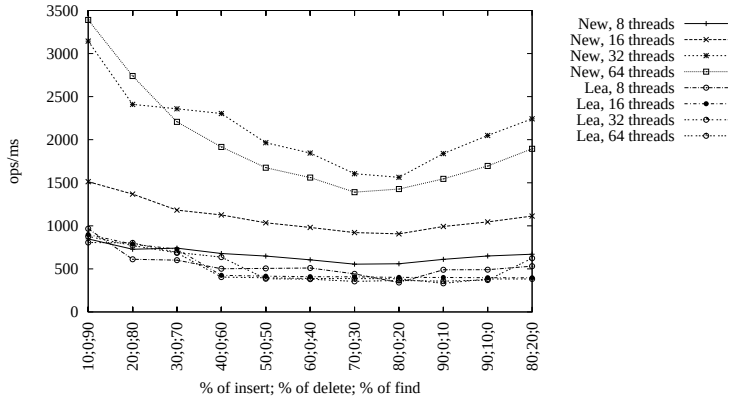


Fig. 10. Varying operation distribution

it involves significantly higher concurrent communication and is thus much more sensitive to the specific layout of threads on the machine and to the load on the shared crossbar. It reaches peak performance at 44 threads when `work` = 0 and scales better to a peak at 56 threads with `work` = 500, that is, when threads perform work between hash table accesses.

—In both cases the performance of our algorithm deteriorates after the peak because of the cost of concurrent communication over the shared crossbar. This contrasts with Lea's algorithm which suffers a much milder deterioration because it never reaches high concurrency levels and its overall performance is limited by the bottlenecks introduced by the shared locks.

—We also measured the performance of both algorithms under low load, when `work` = 3000. Under these conditions Lea's algorithm performed just slightly worse than ours, probably because of our more efficient manipulation of list items and not because of any synchronization related factor.

Figure 10 shows the results of an experiment varying the chosen distribution of

`inserts`, `deletes`, and `finds`, where `work` = 0. Note that our algorithm consistently outperforms Lea's algorithm throughout the full range of tested distributions. We also ran an experiment that varies the load factor in our algorithm. As seen in Figure 9 for the case `work` = 0, the load factor does not affect the performance significantly, and its effect is in any case minimal when compared to those of the thread layout and the overall communication overhead.

Additionally, we tested the robustness of the algorithms under a biased hash function, mimicking conditions in case of a bad choice of a hash function relative to the given data. To do so we generated keys in a non-uniform distribution by randomly turning off 0 to 3 LSBs of randomly chosen integers. Our empirical data shows that our algorithm shows greater robustness: it was slowed down by approximately 7%, while Lea's algorithm's performance decreased by more than 30%. The reason for this is that a biased hash function causes some number of buckets to have many more items than the average load. The locks controlling these buckets in Lea's algorithm are thus contended, causing a performance degradation. This does not happen in the lock-free list used by the new algorithm.

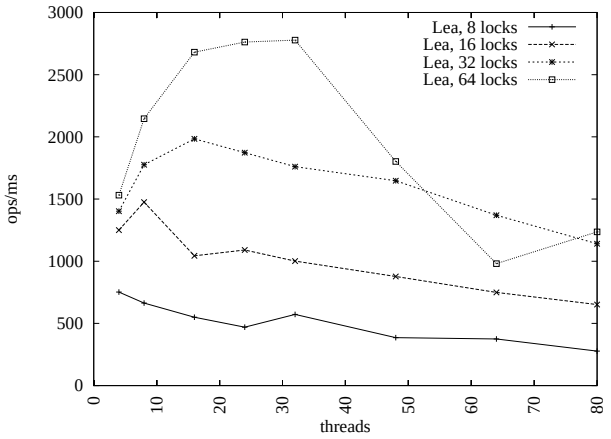


Fig. 11. Lea's algorithm with different concurrency levels

Based on the above results, we conclude that in low-load non-multiprogrammed environments both algorithms offer comparable performance, while under medium to high loads, split-ordered hashing scales better than Lea's algorithm and is thus the algorithm of choice.

5. CONCLUSION

Our paper introduces split-ordered lists and shows how to use them to build resizable concurrent hash tables. We believe the split-order list structure may have broader applications. It might also be interesting to test empirically if a sequential variation of split-ordered hashing will offer an improvement over linear hashing in the sequential case. This follows since splitting buckets in split-ordered hash tables does not require redistribution of individual items among buckets, but rather only the insertion of a dummy node, and in the sequential case the need for the dummy nodes might be avoidable altogether.

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APPENDIX

A. ADDITIONAL CODE

For the purpose of being self contained, this appendix provides the code for the lock-free CAS based ordered list algorithm of Michael [20].

The difficulty in implementing a lock-free ordered linked list is in ensuring that during an insertion or deletion, the adjacent nodes are still valid, i.e. they are still in the list and are still adjacent. Both the implementation of Harris [8] and that of Michael [20] do so by "stealing" one bit from the pointer to mark a node as deleted, and performing the deletion in two steps: first marking the node, and then deleting it. This bit and the *next* pointer are set atomically by the same CAS operation[4]. The `list_find` operation is the most complicated: it traverses through the list, and stops when it reaches an item that is equal-to or greater-than the searched item. If a marked-for-deletion node is encountered, the deletion is completed and the traversal continues. The `list_find` in Michael's scheme thus improves on that of Harris since by completing the deletion immediately when a marked node is encountered it prevents other operations from traversing over marked nodes, that is, ones that have been logically deleted.

The following is a simple lock-free implementation of a shared incrementable (or decrementable) counter using CAS.

[4]Stealing one bit in a pointer in such a manner is straightforward assuming properly aligned memory, and can be achieved with indirection using a "dummy bit node" [1] in languages like the Java™ programming language where stealing a bit in a pointer is a problem. The new Java™ Concurrency Package proposes to eliminate this drawback by offering "tagged" atomic variables.

```
struct MarkPtrType {
  <mark, next>: <bool, NodeType *>
};

struct NodeType {
  key_t key;
  MarkPtrType <mark, next>;
};

/* thread-private variables */
MarkPtrType *prev;
MarkPtrType <pmark, cur>;
MarkPtrType <cmark, next>;

int list_insert(MarkPtrType *head,
                NodeType *node) {
  key = node->key;
  while (1)  {
    if (list_find(head, key) return 0;
    node-><mark,next> = <0,cur>;
    if (CAS(prev, <0,cur>, <0,node>))
      return 1;
  }
}

int list_delete(MarkPtrType *head,
                so_key_t key) {
  while (1) {
    if (!list_find(head, key))
      return 0;
    if (!CAS(&(cur-><mark,next>), <0,next>,
        <1,next>))
      continue;
    if (CAS(prev, <0,cur>, <0,next>))
      delete_node(cur);
    else list_find(head, key);
    return 1;
  }
}

int list_find(NodeType **head, so_key_t key) {
try_again:
  prev = head;
  <pmark,cur> = *prev;
  while (1) {
    if (cur == NULL) return 0;
    <cmark,next> = cur-><mark,next>;
    ckey = cur->key;
    if (*prev != <0,cur>)
      goto try_again;
    if (!cmark) {
      if (ckey >= key)
        return ckey == key;
      prev = &(cur-><mark,next>);
    }
    else {
      if (CAS(prev, <0,cur>, <0,next>))
        delete_node(cur);
      else goto try_again;
    }
    <pmark,cur> = <cmark,next>;
  }
}
```

Fig. 12. Michael's lock free list based sets

```
int fetch-and-inc(int *p) {
  do {
    old = *p;
  } while (!CAS(p, old, old+1);
  return old;
}

int fetch-and-dec(int *p) {
  do {
    old = *p;
  } while (!CAS(p, old, old-1);
  return old;
}
```

Fig. 13. Lock free atomic counter implementation